Applied Mathematics and Computation 227 (2014) 663–671

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

# Global synchronization of uncertain chaotic systems via discrete-time sliding mode control

Ming-Chang Pai

*Department of Automation Engineering, Nan Kai University of Technology, Tsao-Tun, Nantou 54243, Taiwan, ROC*

ARTICLE INFO

*Keywords:*
Discrete-time
Sliding mode control
Chaotic synchronization
Master–slave system
Drive-response system

ABSTRACT

This paper presents a discrete-time sliding mode control scheme for a class of master–slave (or drive-response) chaotic synchronization systems. The proposed scheme guarantees the stability of closed-loop system and achieves the global synchronization between the master and slave systems. The structure of slave system is simple and needs not be identical to the master system. Moreover, the selection of switching surface and the existence of sliding mode have been addressed. Numerical simulations are given to validate the proposed synchronization approach.



## 1. Introduction

Chaotic behavior is an interesting phenomenon appearing nonlinear systems and has been received more and more attentions in the last decades. A chaotic system is a highly complex dynamic nonlinear system. The prominent characteristic of a chaotic system is its extreme sensitivity to initial conditions and the system's parameters, and this makes the problem of chaotic synchronization much more important. In the last few years, chaotic synchronization has applied in vast area of physics and engineering systems such as in chemical reactions, power converter, biological systems, information processing, especially in secure communication [1–3]. Many methods have been developed to realize the problem of the synchronization of chaotic systems including state feedback method [1,2,4–10], the observer method [3,11–15] and output feedback method [16]. However, these methods are developed in continuous-time system. To the best of the author's knowledge, the problem of synchronizing uncertain chaotic systems in discrete-time domain has not been fully investigated and is still open in the literature. This has motivated our research.

On the other hand, using computers or DSP chips to implement the controller has become more and more important nowadays. Therefore, research in discrete-time control has been intensified in recent years, and it is quite natural to extend the technique of continuous control to discrete-time systems. Sliding mode control (SMC) is a nonlinear control approach. The continuous-time SMC is known as a robust method and has attractive features such as fast response, good transient performance, insensitiveness to the matching parameter uncertainties and external disturbances [17,18]. Over the past few years, it has been widely applied to many practical control systems. Several design methods of discrete-time SMC have been proposed in the literature [19–23].

In this paper, a discrete-time SMC scheme is developed to control the synchronization of a class of uncertain chaotic systems in the master–slave (or drive-response) framework. The proposed scheme has the following attractive features: (1) the control design is rather straightforward and ensures the synchronization of the master–slave chaotic systems, (2) the structure of slave system is simple and needs not be identical to the master system, (3) the discrete-time SMC needs not a switching type of control law. Chattering phenomenon and reaching phase are eliminated, (4) the control strategy can be easily

*E-mail address:* pmc@nkut.edu.tw

0096-3003/$ - see front matter 
http://dx.doi.org/10.1016/j.amc.2013.11.075

applied to other dimensional chaotic synchronization problems. The organization of this paper is as follows. Section 2 briefly states the master–slave chaotic synchronization systems. Section 3 provides the proposed discrete-time SMC scheme. Section 4 presents results from numerical simulations. Finally, a conclusion is provided in Section 5.

## 2. System description and problem formulation

Consider a class of chaotic systems described by

$$\dot{x}(t) = Ax(t) + Bg(x,t), \tag{1}$$

where $x(t) \in R^n$ is the state vector, $g(\cdot) \in R^r$ represents the nonlinear function vector that may include unknown parameter perturbations and external disturbances. The constant matrices $A$ and $B$ are known constant matrices of appropriate dimensions.

**Remark 1.** It should be noted that chaotic system (1) is quite popular and has been used in many studies in the literature, such as Lorenz system [4,6,24], Chen chaotic dynamical system [5], Duffing–Holmes system [7], Chua's circuit [8], and Chen–Lee system [25].

The discrete-time representation of system (1) with sample and hold process is given by

$$x(k+1) = \Phi x(k) + \Gamma g(k), \tag{2}$$

where the sampling time is $T$, $\Phi = e^{AT}$ and $\Gamma = \int_0^T e^{A\tau} d\tau B$ [22]. The main objective of this paper is to find a discrete-time sliding mode controller to synchronize the discrete-time chaotic system (2) in the master–slave (or drive-response) framework. To facilitate further development, we make the following assumptions:

**Assumption 1.** The pair $(\Phi, \Gamma)$ is controllable.

**Assumption 2.** The sampling interval $T$ is assumed to be sufficiently small such that the nonlinear function $g(k)$ does not vary too much between consecutive sampling instances. Also, there exists a positive constants $\rho$ such that

$$||g(k)|| \leqslant \rho < \infty. \tag{3}$$

In this paper, for the general class of discrete-time chaotic system (2), the master and slave systems are respectively defined as follows:

$$x_m(k+1) = \Phi x_m(k) + \Gamma g(k), \tag{4}$$

and

$$x_s(k+1) = \Phi x_s(k) + \Gamma u(k), \tag{5}$$

where $x_m(k) \in R^n$ and $x_s(k) \in R^n$ are the master system's state vector and slave system's state vector, respectively, $u(k) \in R^m$ is the control input vector, and $||g(k)|| \leqslant \rho < \infty$.

Let us define the synchronization error $e(k)$ vector as

$$e(k) = x_m(k) - x_s(k). \tag{6}$$

The dynamics of synchronization error between the master and slave systems given in (4) and (5) can be described by

$$e(k+1) = \Phi e(k) + \Gamma g(k) - \Gamma u(k). \tag{7}$$

It is clear that the synchronization problem is replaced by the equivalent problem of stabilizing the synchronization error system (7) using a suitable choice of the control law $u(k)$. In the sequel, using the proposed discrete-time SMC scheme, the asymptotical stability of synchronization error system (7) can be achieved in the sense that $||e(k)|| \to 0$ as $k \to \infty$.

**Remark 2.** The sliding mode characteristics of discrete-time SMC systems are different from those of continuous-time SMC systems. It is noted that the motion of a discrete-time SMC system can approach the switching surface but cannot stay on it in practice. Thus, only the quasi-sliding mode is ensured [19–21].

## 3. Switching surface and discrete-time sliding mode controller design

In this paper, the switching function is defined as follows:

$$s(k) = Ge(k) - G\exp(-\beta k)e(0) - \varepsilon(k), \quad \beta > 0, \tag{8a}$$

$$\varepsilon(k) = \varepsilon(k-1) + G(\Phi + \Gamma K)e(k-1), \quad \varepsilon(0) = 0, \tag{8b}$$

where $G \in R^{m\times n}$ is chosen such that $G\Gamma$ is nonsingular, $K \in R^{m\times n}$ is designed later such that the synchronization error system (7) in the quasi-sliding mode is asymptotically stable, and the exponential term $G\exp(-\beta k)e(0)$ is used to eliminate the reaching phase.

Using the concept of equivalent control, the equivalent control $u_{eq}(k)$ can be found by solving for $s(k) = s(k+1) = 0$

$$u_{eq}(k) = -Ke(k) - (G\Gamma)^{-1}G\exp(-\beta(k+1))e(0) + (G\Gamma)^{-1}G\Gamma g(k) - (G\Gamma)^{-1}\varepsilon(k) \tag{9}$$

or

$$u_{eq}(k) = -Ke(k) - (G\Gamma)^{-1}Ge(k) + (G\Gamma)^{-1}G\Gamma g(k) + M(k) \tag{10}$$

where $G\Gamma$ is assumed to be nonsingular and $M(k) = (G\Gamma)^{-1}G[\exp(-\beta k) - \exp(-\beta(k+1))]e(0)$.

Substituting (10) into (7), the dynamic equation of synchronization error system (7) in the quasi-sliding mode can be obtained as

$$e(k+1) = [\Phi + \Gamma(G\Gamma)^{-1}G + \Gamma K)]e(k) - \Gamma M(k). \tag{11}$$

Neglecting the exponential term $\Gamma M(k)$, it is noted that the synchronization error system (7) in the quasi-sliding mode is insensitive to the nonlinear function. In other words, the controlled system is robust. Eq. (11) can be considered as a linear state feedback problem. The gain matrix $K$ can be designed by using the pole placement method.

**Remark 3.** It is noted that the exponential term $\Gamma M(k)$ in (11) will decay to zero as $k \to \infty$, hence it will not affect the stability of synchronization error system (7) in the quasi-sliding mode.

After designing the switching surface, the next phase is to design the control law such that the quasi-sliding mode is reached and stayed thereafter. Before designing the controllers, we first give a lemma proposed by [20].

**Lemma 1** ([20]). *A necessary and sufficient condition for a discrete sliding mode control system to assure both sliding motion and convergence onto the hyperplane is*

$$||s(k+1)|| < ||s(k)||. \tag{12}$$

*Condition (12) can be further decomposed into the following two inequalities:*

$$s^T(k)[s(k+1) - s(k)] < 0, \tag{13}$$

$$s^T(k)[s(k+1) + s(k)] > 0, \tag{14}$$

*where (13) and (14) are called sliding condition and convergence condition, respectively.*

For the synchronization error system (7), we consider the control law as

$$u(k) = -Ke(k) - (G\Gamma)^{-1}G\exp(-\beta(k+1))e(0) + (G\Gamma)^{-1}Gf(k-1) - (G\Gamma)^{-1}\varepsilon(k) + (G\Gamma)^{-1}\gamma s(k), \tag{15}$$

where the nonlinear function $f(k)$ is defined as $f(k) \triangleq \Gamma g(k)$ which can be estimated through the following relation

$$f(k-1) = e(k) - \Phi e(k-1) + \Gamma u(k-1), \tag{16}$$

and $\gamma$ is a designed parameter.

**Theorem 1.** *Consider the synchronization error system (7) with the proposed control law (15) and the switching function (9). If there exist the matrices G and K such that $\Phi + \Gamma(G\Gamma)^{-1}G + \Gamma K$ is stable, then*

(a) *the quasi-sliding mode condition $||s(k+1)|| < ||s(k)||$ will be satisfied outside the region B, where the region B is defined as* $\Omega_B = \left\{s(k) : ||s(k)|| \leqslant \max_{-1<\gamma<1}\left[\frac{\alpha(k)}{1-\gamma} \quad \frac{\alpha(k)}{1+\gamma}\right]\right\}$ *and* $||G[f(k) - f(k-1)]|| \leqslant \alpha(k)$

(b) *the closed-loop system of synchronization error system (7) with the control law (15) is asymptotically stable.*

**Proof.** First, we prove that the condition $||s(k+1)|| < ||s(k)||$ is satisfied. For this, the proof includes two parts. □

**Part I** (*Sliding condition*). From (7), (8) and (15), the difference between $s(k+1)$ and $s(k)$ can be expressed as

$$s(k+1) - s(k) = G[f(k) - f(k-1)] - (1+\gamma)s(k). \tag{17}$$

Pre-multiplying (17) by $s^T(k)$

$$\begin{aligned} s^T(k)[s(k+1) - s(k)] &= -(1+\gamma)||s(k)||^2 + s^T(k)G[f(k) - f(k-1)] < -(1+\gamma)||s(k)||^2 + ||s(k)||||G[f(k) - f(k-1)]|| \\ &< -(1+\gamma)||s(k)||\left[||s(k)|| - \frac{\alpha(k)}{(1+\gamma)}\right]. \end{aligned}$$

If $||s(k)|| \geqslant \frac{\alpha(k)}{(1+\gamma)}$ and $(1+\gamma) > 0$, then $s^T(k)[s(k+1) - s(k)] < 0$, which implies the sliding condition is achieved.

**Part II** (*Convergence condition*). From (7), (8) and (15), the sum between $s(k+1)$ and $s(k)$ can be expressed as

$$s(k+1) + s(k) = G[f(k) - f(k-1)] + (1-\gamma)s(k). \tag{18}$$

Pre-multiplying (18) by $s^T(k)$

$$\begin{aligned} s^T(k)[s(k+1) + s(k)] &= (1-\gamma)||s(k)||^2 + s^T(k)G[f(k) - f(k-1)] > (1-\gamma)||s(k)||^2 - ||s(k)||\alpha(k) \\ &> (1-\gamma)||s(k)||\left[||s(k)|| - \frac{\alpha(k)}{(1-\gamma)}\right] \end{aligned}$$

If $||s(k)|| \geqslant \frac{\alpha(k)}{(1-\gamma)}$ and $(1-\gamma) > 0$, then $s^T(k)[s(k+1) + s(k)] > 0$, which implies that the convergence condition is achieved.

From Part I, Part II, and Lemma 1, if $||s(k)|| \geqslant \max_{-1<\gamma<1}\left\{\frac{\alpha(k)}{1-\gamma}, \frac{\alpha(k)}{1+\gamma}\right\}$, it concludes $||s(k+1)|| < ||s(k)||$, which indicates that switching function $s(k)$ is decreasing outside $\Omega_B$. Once the quasi-sliding mode condition $||s(k+1)|| < ||s(k)||$ is satisfied, the synchronization error system state trajectories will approach the switching surface in finite time. Next, it is clear to show that the error states $e(k)$ in the quasi-sliding mode are asymptotically stable if there exist the matrices $G$ and $K$ such that $\Phi + \Gamma(G\Gamma)^{-1}G + \Gamma K$ is stable. Hence, the closed-loop system is stable. The proof is completed.

**Lemma 2.** *If the control law (15) is proposed for the synchronization error system (7), then the least upper bound of $||s(k)||$ is equal to $\alpha(k)$.*

**Proof.** From Theorem 1, it is shown that the quasi-sliding mode condition $||s(k+1)|| < ||s(k)||$ will be satisfied outside $\Omega_B$ if the control law (15) is used. Since $\max_{-1<\gamma<1}\left\{\frac{\alpha(k)}{1-\gamma}, \frac{\alpha(k)}{1+\gamma}\right\} \geqslant \alpha(k)$, it can be found that the least upper bound of $\min_{-1<\gamma<1}||s(k)|| = \min\left\{\max_{-1<\gamma<1}\left\{\frac{\alpha(k)}{1-\gamma}, \frac{\alpha(k)}{1+\gamma}\right\}\right\} = \alpha(k)$. The proof is completed. □

**Remark 4.** In general, it is usually desired to have a minimum bound of $\Omega_B$ in order to increase the accuracy of control if the nonlinear function $f(k)$ exists. From Lemma 2, if $\gamma = 0$, the minimum bound of $\Omega_B$ will be obtained. Hence, we select $\gamma = 0$ for the rest of this paper and the control law (15) is modified to be

$$u(k) = -Ke(k) - (G\Gamma)^{-1}G\exp(-\beta(k+1))e(0) + (G\Gamma)^{-1}Gf(k-1) - (G\Gamma)^{-1}\varepsilon(k), \tag{19}$$

where $u(0) = -Ke(0) - (G\Gamma)^{-1}G\exp(-\beta)e(0)$.

It is noted from (19) that there is no switching action in the proposed controllers, which means that chattering phenomenon will never happen. Also, the other advantage of (19) is that the upper bound of nonlinear function $f(k)$ needs not to be known beforehand when the controller is implemented. Hence, it will increase the applicability of the proposed control scheme.

**Theorem 2.** *Consider the master system (4) with Assumptions 1 and 2. If the switching function (8) and the control law (19) are used, and there exist the matrices $G$ and $K$ such that $\Phi + \Gamma(G\Gamma)^{-1}G + \Gamma K$ is stable, then the slave system (5) can globally synchronize the master system (4).*

**Proof.** From Theorem 1, it is clear to show that the proposed control law (19) guarantees the quasi-sliding mode condition $||s(k+1)|| < ||s(k)||$ is satisfied, and the error states $e(k)$ in the quasi-sliding mode are asymptotically stable if there exist the matrices $G$ and $K$ such that $\Phi + \Gamma(G\Gamma)^{-1}G + \Gamma K$ is stable. Therefore, the slave system (5) can globally synchronize the master system (4). The proof is completed. □

## 4. Illustrative examples

To demonstrate the effectiveness of the proposed synchronization scheme, simulations results of three-dimensional Lorenz system and Chen–Lee system are given in this section.

**Example 1** (*Lorenz system*). The dynamics of Lorenz system [24] can be transformed into the form of system (1) as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -a & a & 0 \\ c & -1 & 0 \\ 0 & 0 & -b \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -x_1x_3 \\ x_1x_2 \end{bmatrix}, \tag{20}$$

where the variables $x_1$, $x_2$ and $x_3$ in (20) represent measures of fluid velocity and horizontal and vertical temperature variations, respectively, and the parameters $a$, $b$, and $c$ are positive parameters representing the Prandtl number, a geometric factor, and the Rayleigh number, respectively. The dynamics of discrete-time chaotic Lorenz systems with sample time $T = 0.001$ s, $a = 10$, $b = \frac{8}{3}$, and $c = 28$ are given by

$$x_m(k+1) = \Phi x_m(k) + \Gamma g(k) = \begin{bmatrix} 0.990 & 0.010 & 0 \\ 0.028 & 0.999 & 0 \\ 0 & 0 & 0.997 \end{bmatrix} x_m(k) + \begin{bmatrix} 0 & 0 \\ 0.001 & 0 \\ 0 & 0.001 \end{bmatrix} \begin{bmatrix} -x_{m1}(k)x_{m3}(k) \\ x_{m1}(k)x_{m2}(k) \end{bmatrix}. \tag{21}$$

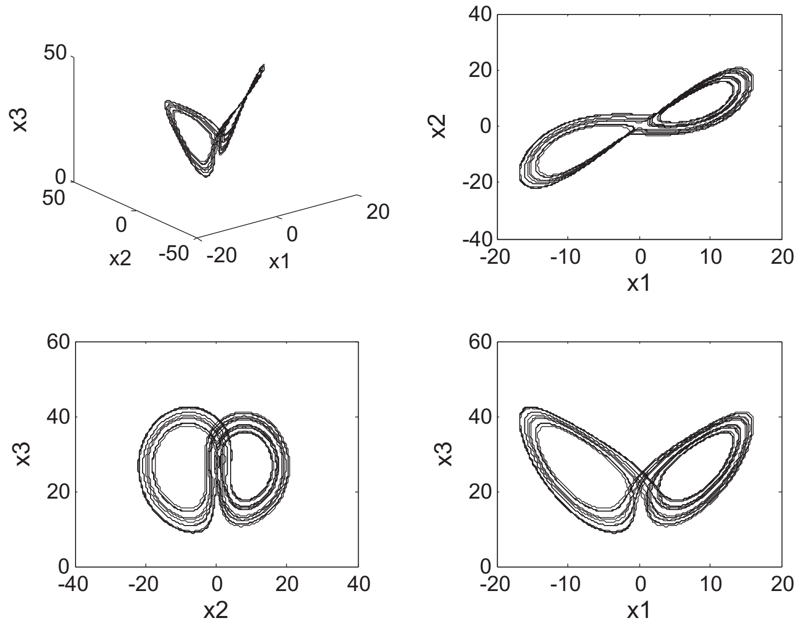


**Fig. 1.** The chaotic attractor of Lorenz system.

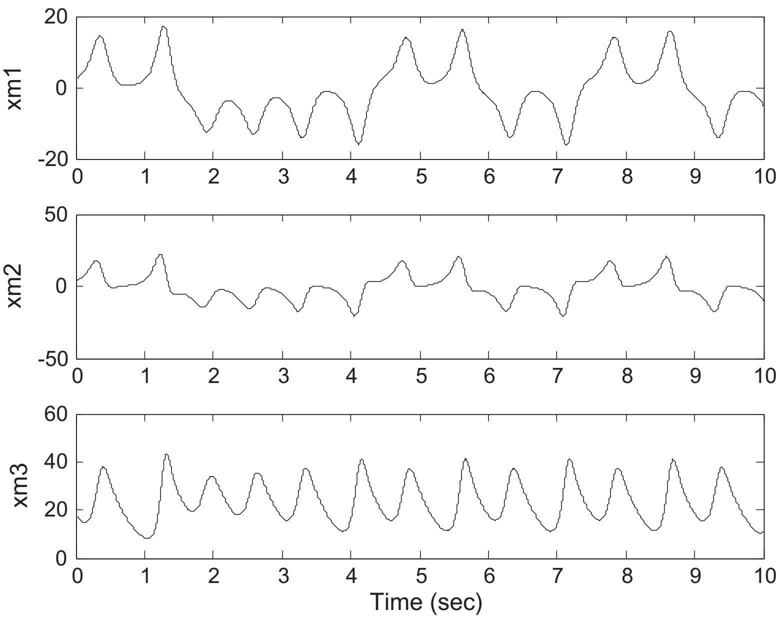


**Fig. 2.** The chaotic motions of Lorenz system (the mater system).

The chaotic figures of Lorenz system (21) are shown in Fig. 1 and display a 2-scroll chaotic attractor. For the master system (21), the dynamics of the slave system is given by

$$x_s(k+1) = \Phi x_s(k) + \Gamma u(k) = \begin{bmatrix} 0.990 & 0.010 & 0 \\ 0.028 & 0.999 & 0 \\ 0 & 0 & 0.997 \end{bmatrix} x_s(k) + \begin{bmatrix} 0 & 0 \\ 0.001 & 0 \\ 0 & 0.001 \end{bmatrix} u(k). \tag{22}$$

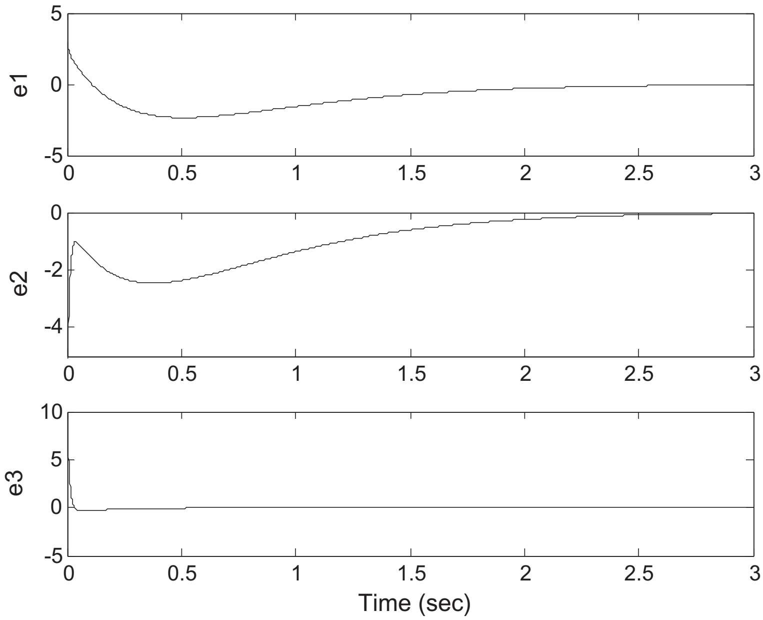


**Fig. 3.** Synchronization errors between master and slave systems.

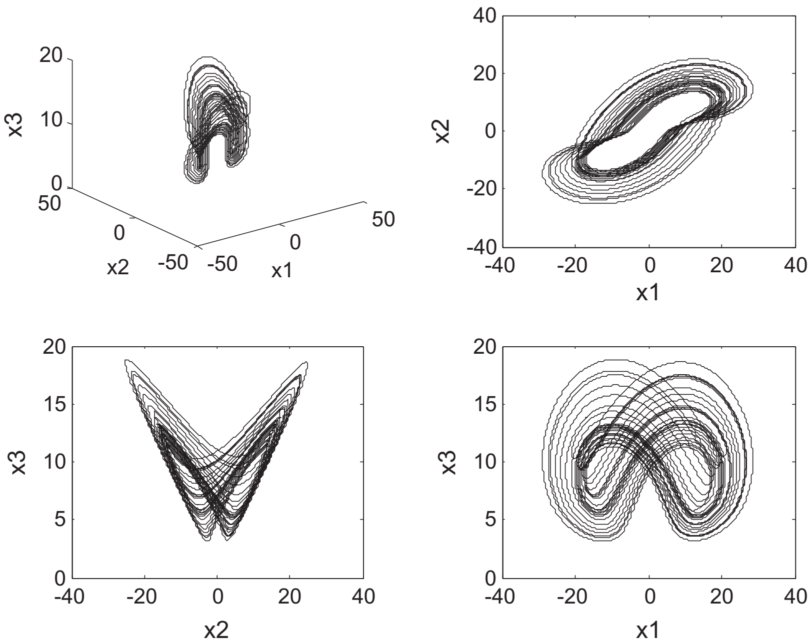


**Fig. 4.** The chaotic attractor of Chen–Lee system.

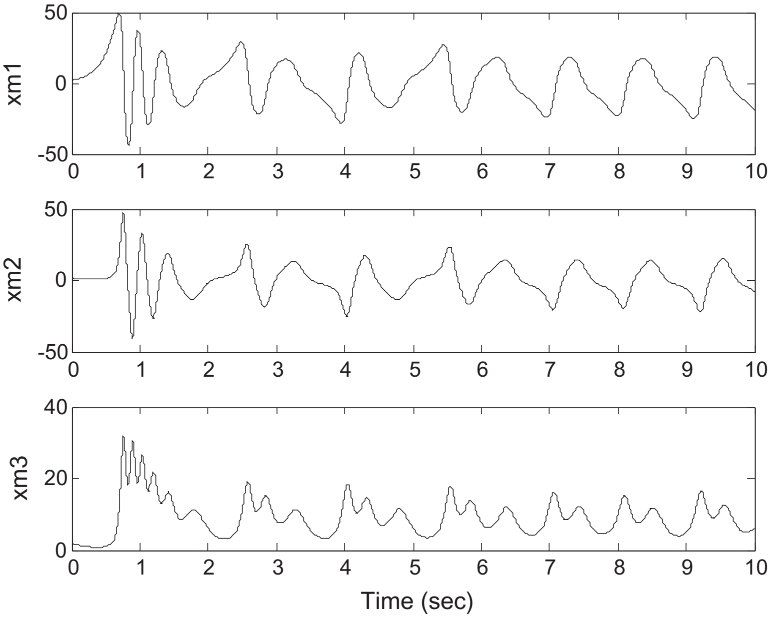


**Fig. 5.** The chaotic motions of Chen–Lee system (the mater system).

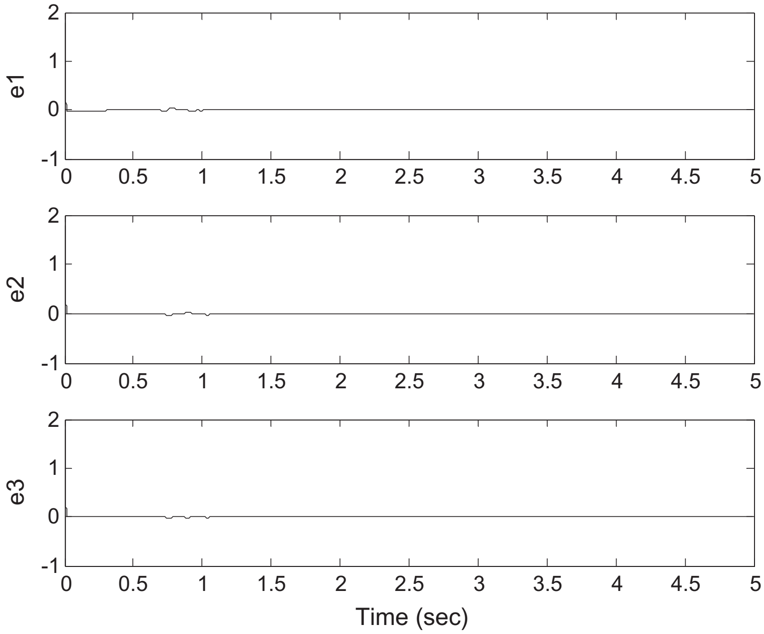


**Fig. 6.** Synchronization errors between master and slave systems (Pai's method).

The initial states of the master system (21) and the slave system (22) are $x_m(0) = [2 \quad 3.5 \quad 18.4]^T$ and $x_s(0) = [-1 \quad 10.2 \quad 8.3]^T$, respectively. Using the proposed method, the constant matrices $G$ and $K$ in the switching function (8) are designed, respectively as $G = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $K = \begin{bmatrix} -33.4 & -994.6 & 0 \\ 0 & 0 & -1002.7 \end{bmatrix}$ such that $G\Gamma$ is nonsingular and eigenvalues of $\Phi + \Gamma(G\Gamma)^{-1}G + \Gamma K$ are located at $[0.996 \quad 0.997 \quad 0.998]$. The parameter $\beta$ in controller (19) is selected as 0.1. The simulation results are shown in Figs. 2 and 3. Fig. 2 shows chaotic motions (or state trajectories) of the master system. Fig. 3 depicts the synchronization error of state variables between the master system (21) and the slave system (22). It is clearly shown that the error states converge to zero asymptotically.

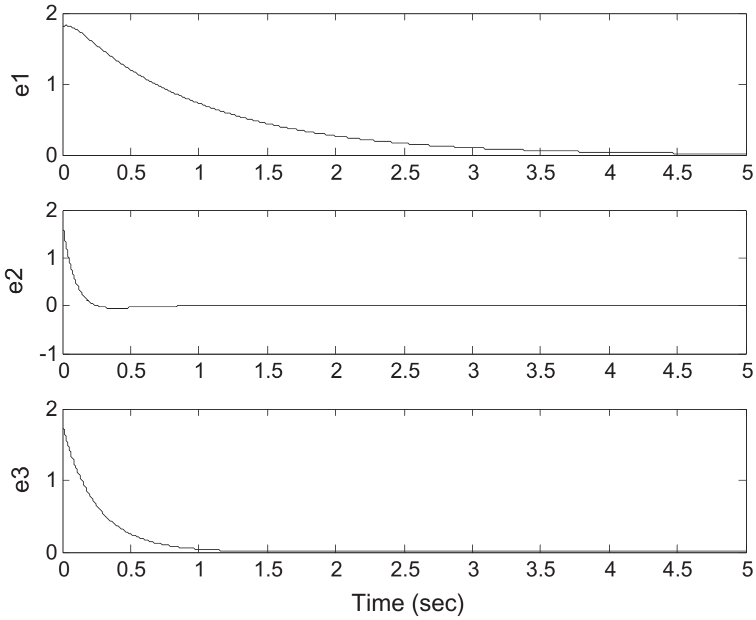


**Fig. 7.** Synchronization errors between master and slave systems (Chen's method [6]).

**Example 2** (*Chen–Lee system*). The dynamics of Chen–Lee system [25] can be represented as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_1 & 0 \\ 0 & 0 & c_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -x_2x_3 \\ x_1x_3 \\ \frac{1}{3}x_1x_2 \end{bmatrix}, \tag{23}$$

Where $x_1, x_2$ and $x_3$ in (23) are the state variables and $a_1, b_1, c_1$ are three system parameters. The dynamics of discrete-time chaotic Chen–Lee systems with sample time $T = 0.001$ second, $a_1 = 5$, $b_1 = -10$, and $c_1 = -3.8$ are given by

$$x_m(k+1) = \Phi x_m(k) + \Gamma g(k) = \begin{bmatrix} 1.005 & 0 & 0 \\ 0 & 0.990 & 0 \\ 0 & 0 & 0.996 \end{bmatrix} x_m(k) + \begin{bmatrix} 0.001 & 0 & 0 \\ 0 & 0.001 & 0 \\ 0 & 0 & 0.001 \end{bmatrix} \begin{bmatrix} -x_{m2}(k)x_{m3}(k) \\ x_{m1}(k)x_{m3}(k) \\ \frac{1}{3}x_{m1}(k)x_{m2}(k) \end{bmatrix}. \tag{24}$$

The chaotic figures of Chen–Lee systems (24) are shown in Fig. 4 and display a 2-scroll chaotic attractor. For the master system (24), the dynamics of the slave system is given by

$$x_s(k+1) = \Phi x_s(k) + \Gamma u(k) = \begin{bmatrix} 1.005 & 0 & 0 \\ 0 & 0.990 & 0 \\ 0 & 0 & 0.996 \end{bmatrix} x_s(k) + \begin{bmatrix} 0.001 & 0 & 0 \\ 0 & 0.001 & 0 \\ 0 & 0 & 0.001 \end{bmatrix} u(k). \tag{25}$$

The initial states of the master system (24) and the slave system (25) are $x_m(0) = [2 \quad 2 \quad 2]^T$ and $x_s(0) = [0.2 \quad 0.2 \quad 0.2]^T$, respectively. Using the proposed method, the constant matrices $G$ and $K$ in the switching function (8) are designed, respectively as

$$G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad K = \begin{bmatrix} -1007.5 & 0 & 0 \\ 0 & -997.0 & 0 \\ 0 & 0 & -999.1 \end{bmatrix},$$

such that $G\Gamma$ is nonsingular and eigenvalues of $\Phi + \Gamma(G\Gamma)^{-1}G + \Gamma K$ are located at $[0.995 \quad 0.998 \quad 0.999]$. The parameter $\beta$ in controller (19) is selected as 0.1. The simulation results are shown in Figs. 5 and 6. Fig. 5 shows chaotic motions (or state trajectories) of the master system. Fig. 6 shows the synchronization error of state variables between the master system (24) and the slave system (25). It can be seen that the synchronization of the two systems is completed after 1.3 s.

In the following, we will make a comparison between the proposed method and the method by [6]. In this method, the structure of slave system needs be identical to the master system. Following the design procedure in [6], the synchronization errors are shown in Fig. 7. From Figs. 6 and 7, it is clearly shown that the proposed method provides faster response and better synchronization performance than the Chen's method. Furthermore, one of advantages in the proposed method is that the structure of slave system is simple and needs not be identical to the master system, which is more general and flexible for the synchronization problem for a class of uncertain chaotic systems. Therefore, our strategy outperforms the method proposed in [6]. From these results, we can see that the proposed scheme yields good synchronization for uncertain chaotic systems.

## 5. Conclusions

In this paper, a discrete-time SMC scheme has been proposed for synchronization of a class of chaotic systems. It has been shown that the proposed control scheme ensures the stability of synchronization error dynamics, and provides good chaotic synchronization between the master and slave systems. The control design is rather straightforward and easy to implement for chaotic synchronization. The discrete-time SMC needs not a switching type of control law. Chattering phenomenon and reaching phase are eliminated. Moreover, the control strategy can be easily applied to other dimensional chaotic synchronization problems. Numerical simulation has confirmed the validity of the proposed synchronization scheme.

## References

[1] T.L. Liao, S.H. Tsai, Adaptive synchronization of chaotic systems and its application to secure communications, Chaos Solitons Fractals 11 (2000) 1387–1396.
[2] M. Feki, An adaptive chaos synchronization scheme applied to secure communication, Chaos Solitons Fractals 18 (2003) 141–148.
[3] M. Chen, D. Zhou, Y. Shang, A sliding mode observer based secure communication scheme, Chaos Solitons Fractals 25 (2005) 573–578.
[4] S.K. Yang, C.L. Chen, H.T. Yau, Control of chaos in Lorenz system, Chaos Solitons Fractals 13 (2002) 767–780.
[5] M.T. Yassen, Chaos control of Chen chaotic dynamical system, Chaos Solitons Fractals 15 (2003) 271–283.
[6] H.K. Chen, Global chaos synchronization of new chaotic systems via nonlinear control, Chaos Solitons Fractals 23 (2005) 1245–1251.
[7] H.T. Yau, Design of adaptive sliding mode controller for chaos synchronization with uncertainties, Chaos Solitons Fractals 22 (2004) 341–347.
[8] J.J. Yan, M.L. Hung, T.Y. Chiang, Y.S. Yang, Robust synchronization of chaotic systems via adaptive sliding mode control, Phys. Lett. A 356 (2006) 220–225.
[9] H.H. Chen, Global synchronization of chaotic systems via linear balanced feedback control, Appl. Math. Comput. 186 (2007) 923–931.
[10] Y.C. Hung, T.L. Liao, J.J. Yan, Adaptive variable structure control for chaos suppression of unified chaotic systems, Appl. Math. Comput. 209 (2009) 391–398.
[11] C. Hua, X. Guan, X. Li, P. Shi, Adaptive observer-based control for a class of chaotic systems, Chaos Solitons Fractals 22 (2004) 103–110.
[12] M. Chen, D. Zhou, Y. Shang, Synchronizing a class of uncertain chaotic systems, Phys. Lett. A 337 (2005) 384–390.
[13] S.H. Mahboobi, M. Shahrokhi, H.N. Pishkenari, Observer-based control design for three well-known chaotic systems, Chaos Solitons Fractals 29 (2006) 381–392.
[14] F.M.M. Kakmeni, S. Bowong, C. Tchawoua, Nonlinear adaptive synchronization of a class of chaotic systems, Phys. Lett. A 355 (2006) 47–54.
[15] F.L. Zhu, Full-order and reduced-order observer-based synchronization for chaotic systems with unknown disturbances and parameters, Phys. Lett. A 372 (3) (2008) 223–232.
[16] Y.Y. Hou, T.L. Liao, J.J. Yan, $H_\infty$ synchronization of chaotic systems using output feedback control design, Phys. A 379 (2007) 81–89.
[17] B. Draženović, The invariance conditions in variable structure systems, Automatica 5 (3) (1969) 287–295.
[18] V.I. Utkin, Sliding mode systems with sliding modes, IEEE Trans. Autom. Control 22 (2) (1977) 212–222.
[19] D. Milosavljevic, General conditions for the existence of a quasi-sliding mode on the switching hyperplane in discrete variable structure systems, Autom. Remote Control 46 (1985) 307–314.
[20] S.Z. Sarpturk, Y. Istefanopulos, O. Kaynak, On the stability of discrete-time sliding mode control systems, IEEE Trans. Autom. Control 32 (10) (1987) 930–937.
[21] W.B. Gao, Y. Hung, A. Homaifa, Discrete-time variable structure control systems, IEEE Trans. Ind. Electron. 42 (2) (1995) 117–122.
[22] K.D. Young, V.I. Utkin, Ü. Özgüner, A control engineer's guide to sliding mode control, IEEE Trans. Control Syst. Technol. 7 (3) (1999) 328–342.
[23] K. Abidi, J.X. Xu, Y. Xinghuo, On the discrete-time integral sliding-mode control, IEEE Trans. Autom. Control 52 (4) (2007) 709–715.
[24] E.N. Lorenz, Deterministic nonperiodic flow, J. Atmos. Sci. 20 (1963) 130–141.
[25] H.K. Chen, C.I. Lee, Anti-control of chaos in rigid body motion, Chaos Solitons Fractals 21 (2004) 957–965.